SECTION A: Attempt **all** questions. **(55marks)**

1. Use the addition and subtraction formulas to find the exact value of: **(5 marks)**

   i. $\sin 75^0$

   ii. $\cos\frac{13\pi}{6}$

2. Find the derivative of $f(x) = sin^{-1}(1 - x^2)$ **(3 marks)**

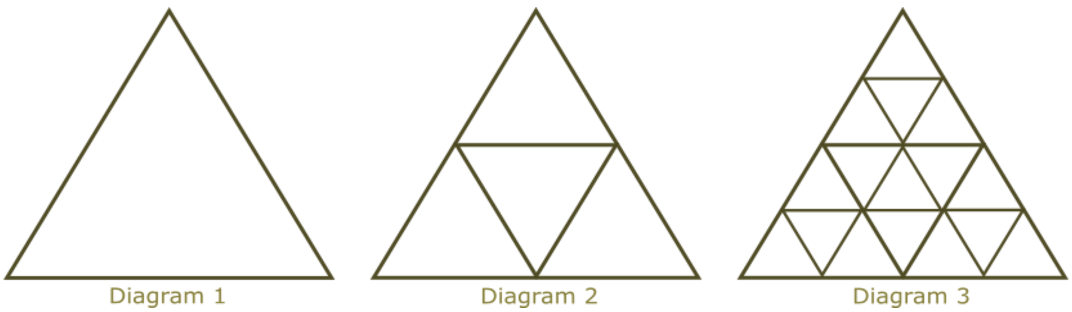
3. Evaluate $\lim\limits_{x\to 1} \frac{x^2-1}{sin^{-1}(1-x)}$ **(4 marks)**

4. The expression $\cos 2x + 2\sin^2 x$ is equivalent to:

   i) 1 $\quad ii)\ sinx \quad iii)\ cos^2x \quad iv)\ \frac{1}{2}\tan 2x$ **(1.5mark)**

5. Solve the following equation: $2(5^{2x}) - 5^x = 6$ **(3 marks)**

6.



The diagrams above show a growing fractal of triangles. The sides of the largest equilateral triangle in each diagram are of length 1 meter.

In the second diagram there are four triangles each with sides of length $\frac{1}{2}$ metre.

In the third diagram there are 16 triangles each with sides of length $\frac{1}{4}$ metre.

(a) Complete this table for more diagrams. **(2 marks)**

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	...	Diagram *n*
Length of Side	1	$\frac{1}{2}$	$\frac{1}{4}$			
Power of 2	2^0	2^{-1}	2^{-2}			

(b) Complete this table for the number of the smallest triangles in each diagram. **(2marks)**

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	…	Diagram n
Number of smallest triangles	1	4	16			
Power of 2	2^0	2^2	2^4			

7. Use the Newton-Raphson method twice to approximate the positive real solution of the equation $x^3 - x - 1 = 0$, correct to three decimal places **(4 marks)**
8. If the first and tenth terms of a geometric sequence are 1 and 4 respectively, find the nineteenth term.

   (4 marks)
9. Find the fundamental period of $f(x) = \tan(\frac{x+1}{2})\sin(\frac{2x+1}{5})$ (4 marks)
10. Insert 5 arithmetic means between 2 and 20 **(5 marks)**
11. Solve the equation: $3sinx + \sqrt{3}cosx = 3$ **(4.5 marks)**
12. According to United data, the world population at the beginning of 1975was approximately 4 billion and growing at a rate of about 2% per year. Assuming an exponential growth model $P = P_0e^{rt}$, estimate the world population at the beginning of the year 2020. **(4 marks)**
13. Write the vector $v(1,-2,5)$ as a linear combination of the vectors $e_1 = (1,1,1)$, $e_2 = (1,2,3)$ and $e_3 = (2,-1,1)$ **(4 marks)**
14. Solve in IR the equation: $\log_x 5 = \log_5 x$
15. Show that the equation $x^3 - 3x - 12 = 0$ has a root between 2 and 3 . Hence use linear interpolation to approximate that root. **(5 marks)**

<u>SECTION B:</u> Attempt **only three** chosen questions. **(45marks)**

16. a. Prove that $\cot a - \tan a = 2\cot 2a$ **(5 marks)**

 b. Transform in sum: $\sin 4x \cos 11x$ **(5 marks)**

 c. Evaluate the limit: $\lim\limits_{x\to 3} \dfrac{\sin\left(x^2 - 3x\right)}{x^2 - 9}$ **(5 marks)**

17. In the IR-vector space IR^3, the subsets U and W are defined by $U = \left\{(x, y, z): x - y - 2z = 0\right\}$ and $W = \left\{(x, y, z): x = 3z\right\}$. Show that U and W are subspaces of IR^3. Find also a basis and the dimension of each subspace. **(15 marks)**

18. a) Solve the following equation for t: $4\sin^2\left(\dfrac{t}{3}\right) - 3\sin\left(\dfrac{t}{3}\right) = 1$ **(7.5 marks)**

 b) Solve the system in IR: $\begin{cases} \ln xy = 3 \\ \ln x \cdot \ln y = 2 \end{cases}$ **(7.5 marks)**

19. A curve C is defined by the parametric equations $x = t^2$ and $y = t^3 - 3t$.

 a. Show that C has two tangents at the point $(3,0)$ and find their equations.

 b. Find the points on C where the tangent is horizontal or vertical.

 c. Determine where the curve rises and falls and where it is concave upward or downward **(15 marks)**

20. a. Find the domain of the following function: $f(x) = \dfrac{1}{x} + \sin^{-1} 2x$

b. Find three numbers in a geometric sequence whose sum is $\frac{13}{12}$ and product is -1.

c. The amount, $A(t)$ gram, of a radioactive material in a sample after t years is given by $A(t)=80\left(2^{-\frac{t}{100}}\right)$.

i. Find the amount of material in the original sample.

ii. Calculate the half-life of the material.

iii. Calculate the time taken for the material to decay to 1 gram. **(15 marks)**

END